11.10

BINOMIAL TH'M

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n$$

WHERE $\binom{p}{n} = \dfrac{p(p-1)(p-2)\cdots(p-n+1)}{n!}$ ← n FACTORS ← n FACTORS

RECALL: $\binom{p}{0} = 1 \quad \binom{p}{1} = p$

AND $\binom{p}{n+1} = \dfrac{p-n}{n+1}\binom{p}{n}$

$$(1+x)^p = \binom{p}{0}x^0 + \binom{p}{1}x^1 + \binom{p}{2}x^2 + \binom{p}{3}x^3 + \cdots$$

$$= 1 + px + \frac{p-1}{2}px^2 + \frac{p(p-1)(p-2)}{3\cdot 2\cdot 1}x^3 + \cdots$$

$$= 1 + \frac{px}{1!} + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \frac{p(p-1)(p-2)(p-3)}{4!}x^4 + \cdots$$

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} \leftarrow p=\tfrac{1}{2}$$

$$= 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}x^4 + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}x^3 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}x^4 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots$$

$$\downarrow \quad \frac{-\frac{5}{2}}{4}\cdot\frac{1}{16} \qquad -\frac{5}{8}\cdot\frac{1}{16}$$

$$= 1 + \sum_{n=\cancel{0}\,1}^{\infty}(-1)^{n+1}\frac{\frac{(2n-2)!}{2^n 2^{n-1}(n-1)!}}{n!}x^n = \boxed{1 + \sum_{n=\cancel{0}\,1}^{\infty}(-1)^{n+1}\frac{(2n-2)!}{2^{2n-1}(n-1)!\,n!}x^n}$$

CONSIDER $1\cdot 3\cdot 5\cdots(2n-3)$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdots(2n-3)(2n-2)}{2 \quad 4 \quad \cdots \quad (2n-2)} \leftarrow (2n-2)!$$

$$= \frac{(2n-2)!}{(2\cdot 1)(2\cdot 2)(2\cdot 3)\cdots(2\cdot(n-1))}$$

$$2^{n-1} \rightarrow \qquad (n-1)!$$

$$= \frac{(2n-2)!}{2^{n-1}(n-1)!}$$

$$\frac{1}{(1-x)^2} = (1+\underline{\underline{(-x)}})^{-2} \qquad p=-2$$

$$= \sum_{n=0}^{\infty} \binom{-2}{n} (-x)^n$$

$$= 1 - 2x + \frac{(-2)(-3)}{2!}x^2 + \frac{(-2)(-3)(-4)}{3!}x^3 + \frac{(-2)(-3)(-4)(-5)}{4!}x^4 + \cdots$$

$$\cancel{= 1 - 2x}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)!}{n!} x^n$$

$$= \sum_{n=0}^{\infty} (-1)^n (n+1) x^n$$

FIND THE T.S. FOR arcsin x ABOUT x=0

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} = \left(1+(-x^2)\right)^{-\frac{1}{2}}$$

$$\arcsin x = \int \left(1+(-x^2)\right)^{-\frac{1}{2}} dx + C$$

$$-\frac{1+2n-2}{2} = -\frac{2n-1}{2}$$

$$\left(1+(-x^2)\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-x^2)^n$$

$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots\left(-\frac{1}{2}-n+1\right)}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{\cancel{(-1)^n}(2n)!}{2^{2n}(n!)^2} \cancel{(-1)^n} x^{2n}$$

$$= (-1)^n \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n \cdot n!}$$

USE n+1 FOR n IN PRIOR SIMPLIFICATION

$$= \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} x^{2n}$$

$$= \frac{(-1)^n \frac{(2n)!}{2^n n!}}{2^n \cdot n!}$$

$$\arcsin x = \int \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} x^{2n} dx + C$$

$$= \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}$$

$$= \sum_{n=0}^{\infty} \int \frac{(2n)!}{2^{2n}(n!)^2} x^{2n} dx + C$$

$$= \sum_{n=0}^{\infty} \frac{(2n)!}{(2n+1)2^{2n}(n!)^2} x^{2n+1} + C \longrightarrow \arcsin 0 = 0 = C$$

$$= \sum_{n=0}^{\infty} \frac{(2n)!}{(2n+1)2^{2n}(n!)^2} x^{2n+1}$$